

Percentages

Expressing a Quantity as a Percentage of Another

Percentage is a measure of **proportion** - one quantity in relation to another.

It can be thought of as a very specific type of fraction, as percentages are **amounts per hundred**.

Expressing a quantity as a percentage of another is easy when the total amount is 100:

74 out of 100 pupils prefer sleeping to school.

$\frac{74}{100}$ is 74%. Easy.

In all other cases, convert the fraction to a decimal and multiply by 100.

$\frac{17}{56}$ YouTube videos you watched featured an American. What percentage is this?

$$\frac{17}{56} = 0.304 \text{ (to 3d.p.)} \quad 0.304 \times 100 = 30.4\%$$

Finding a Percentage of a Quantity

(**Remember:** The word '**of**' is just an alternative word for **multiply**)

Examples:

$$\begin{aligned} \text{a) } 42\% \text{ of } 563\text{g} &= \frac{42}{100} \times 563 \\ &= 0.42 \times 563 \\ &= 236.46\text{g} \end{aligned}$$

Don't forget $\frac{42}{100}$ is the same as $42 \div 100$

$$\begin{aligned} \text{b) } 125\% \text{ of } £3500 &= \frac{125}{100} \times 3500 \\ &= 1.25 \times 3500 \\ &= £4375 \end{aligned}$$

As you get better at calculating percentages you might find you can miss this stage out.

Calculating Percentage Change

The change in amount could be an **increase** or **profit**, or a **decrease** or **loss**.

The meaning of your calculation will depend on the context.

$$\text{percentage change} = \frac{\text{change}}{\text{original value}} \times 100$$

Example:

The first episode of the new X Factor series had 7.33 million viewers. The second episode had 6.88 million viewers.

What was the percentage change in the number of viewers?

The change in the number of viewers is 7.33 million – 6.88 million = 0.45 million **fewer** people than the week before.

So, the percentage **decrease** is $\frac{0.45}{7.33} \times 100 = \mathbf{6.14\%}$ (to 2d.p.)

Increasing or Decreasing a Quantity by a Given Percentage

Increase

The original value is 100%. A percentage increase will **add** to this amount.

100% (original value) + **percentage increase** (C)

We **can** just find the percentage change and add it on to the original value.

But.. here is the **one-step shortcut**:

Result of percentage increase = original amount $\times (1 + \frac{C}{100})$

All we have done is included the original amount (the '1') in our calculation at the beginning to save us time adding it on later.

Remember, when calculating with percentages we have to convert to their decimal form first ($\div 100$)

$$\frac{100}{100} = 1 \text{ (original value)}$$

$$\frac{C}{100} = C \div 100 \text{ (percentage change)}$$

Decrease

The original value is 100%. A percentage decrease will **subtract** from this amount.

100% (original value) - **percentage increase** (C)

We **can** just find the percentage change and **subtract** it from the original value.

But... here is the **one-step shortcut**:

Result of percentage decrease = original amount $\times (1 - \frac{C}{100})$

Like before, the original amount (the '1') is included in our calculation at the beginning to save us time subtracting it later.

Examples

Increase

My savings account has £432. I receive 4% interest. How much money do I have now?

The percentage increase is 4% so $C = 4$

$$\begin{aligned} \text{Resulting amount} &= 432 \times (1 + \frac{4}{100}) \\ &= 432 \times (1 + 0.04) \\ &= 432 \times 1.04 \\ &= \text{£449.28} \end{aligned}$$

Decrease

My monthly pay is £2800 (lucky me!) but I am charged 8% for my pension contribution (yawn...).

How much does this leave me with?

The percentage decrease is 8% so $C = 8$

$$\begin{aligned} \text{Resulting amount} &= 2800 \times (1 - \frac{8}{100}) \\ &= 2800 \times (1 - 0.08) \\ &= 2800 \times 0.92 \\ &= \text{£2576} \\ &\text{(...huge sigh of disappointment)} \end{aligned}$$

Finding the Original Value

Sometimes we are given the percentage change and the new value and we have to find the **original value**.

Very important: Is the percentage change (C) an **increase** or a **decrease**?

A percentage increase will **add** to the original amount.

Original value (100%) + **percentage change**

So: **original amount** = **new amount** $\div$ (100% + C)

$$\text{original amount} = \frac{\text{new amount}}{1 + \frac{C}{100}}$$

A percentage decrease will be **subtracted** from the original amount.

Original value(100%) - **percentage change**

original amount = **new amount** $\div$ (100% - C)

$$\text{original amount} = \frac{\text{new amount}}{1 - \frac{C}{100}}$$

Examples

Percentage Increase

The viewing figures for Gogglebox increase by 6% from episode 1 to episode 2.

By episode 2, the ratings are 8.32 million. How many people watched episode 1?

A percentage **change** will add to the original amount.

$$\begin{aligned}\text{Original amount} &= \frac{8.32}{1 + 0.06} \\ &= \frac{8.32}{1.06}\end{aligned}$$

= 7.85 million (to 2d.p.) viewers watched episode 1.

Percentage Decrease

The viewing figures for Eastenders decrease by 12% from Monday's episode to Tuesday's episode.

On Tuesday, the ratings were 6.42 million. What were the ratings on Monday?

A percentage **change** will subtract from the original amount.

$$\begin{aligned}\text{Original amount} &= \frac{6.42}{1 - 0.12} \\ &= \frac{6.42}{0.88}\end{aligned}$$

= 7.30 million (to 2d.p.) viewers watched Monday's episode.

Remember, when calculating with percentages we have to convert to their decimal form first ($\div 100$)

$$\frac{100}{100} = 1 \text{ (original value)}$$

$$\frac{C}{100} = C \div 100 \text{ (percentage change)}$$