

Ready to progress?



I can show my understanding of how the number line extends to include negative numbers.
I can understand and use the inequality symbols $<$ for less than and $>$ for greater than.
I can use my understanding of negative numbers to solve simple real-life problems.
I can carry out addition and subtraction involving negative numbers.



I can carry out multiplications involving negative numbers.

Review questions

- 1 The keel (bottom) of a cruise ship is 7 m below the surface of the water. The deck is 27 m above the water.
What is the total height, from deck to keel?
- PS 2 Harry is on the 29th floor of a skyscraper. He goes up 18 floors and then down 23 floors. He wants to go to the 2nd floor.
How many floors does he need to go down now?
- FS 3 a Alisha opens a bank account with a deposit of £400. In the next two weeks she takes out £170, deposits £130 and takes out £120.
How much is in her account now?
b Next, Alisha deposits £60 and takes out £170.
How much is in her account now?
- PS 4 Karen and Geza have parked their car on level -5 of the shopping centre car park. They take the lift to the shops on level +6, then Karen realises she has left her purse in the car. She goes back to the car to get it. Then she returns to the shops and meets Geza on level +4.
How many levels has Karen travelled through altogether?
- 5 This is a bank statement.

Statement number: 10		Account number 14628769		
Date	Details	Debit (£)	Credit (£)	Balance (£)
31-01-2014	Opening balance			827.54
01-02-2014	Wage		252.71	
03-02-2014	Cash withdrawal	130.00		
05-02-2014	Transfer to savings account	250.00		
08-02-2014	Wage		252.71	
28-02-2014	Closing balance			

- a Copy the statement and complete the balance column.
- b Explain the difference between a debit and a credit.
- 6 The Mariana Trench in the Pacific Ocean has a maximum depth of 10911 metres below sea level. The summit of Mount Everest is 8848 metres above sea level.
a How much higher than the base of the trench is the summit of Mount Everest?
b If Mount Everest was set in the deepest part of the trench, how far would its summit be below sea level?

PS

PS

- 7 In a popular TV programme, each of two teams has to buy three items at an antiques fair.

The items are sold at auction and the team that makes more money wins and keeps any profit they make.

Copy and complete each team's score sheet.

- a Red team

Item	Buying price (£)	Selling price (£)	Profit (£)
Silver dish	59		-18
Umbrella		47	+14
Toy car	45	55	
Total	137		

- b Blue team

Item	Buying price (£)	Selling price (£)	Profit (£)
Necklace		55	+17
Doll	49	85	
Watch	110		-52
Total	197		

- c Which team won and by how much?
- d Teams can go for a bonus buy, chosen by an expert.
The Red team bonus buy was bought for £33 and sold for £25.
The Blue team bonus buy was bought for £21 and sold for £29.
Copy and complete this table to show who would win in each case.

	Red team without the bonus buy	Red team with the bonus buy
Blue team without the bonus buy		
Blue team with the bonus buy		

PS

PS

- 8 Work out the answers.

- | | | | |
|--------------------|--------------------|----------------------|-----------------------|
| a $5 \times (-1)$ | b -3×9 | c -8×-7 | d -11×12 |
| e $-9 \times (-7)$ | f $8 \times (-3)$ | g $5 \times (-12)$ | h $-12 \times (-8)$ |
| i -23×-1 | j -14×100 | k $-30 \times (-10)$ | l $(-8) \times (-90)$ |

Problem solving

Where in the world?

A Where shall we go?

David and Hannah were discussing where they could go for their honeymoon in the summer.

They were interested in the places listed in the table below.

They worked out the time difference from the UK for each one.

- 1 Which place is furthest behind the UK, in terms of time?
- 2 What is the time difference between Reykjavik and Amsterdam?
- 3 What is the time difference between San Francisco and New York?
- 4 What time will it be in Bangkok when it is 10:00 am in the UK?
- 5 What time is it in New Delhi when it is 6:00 pm in New York?

City	Difference from UK time (hours)
Amsterdam	+1
Bangkok	+7
Hong Kong	+8
New Delhi	+5:30
New York	-5
Reykjavik	0
San Francisco	-8
Sydney	+11

B How shall we get there?

The couple decided to go to Sydney, so they looked up some flight times and prices.

David and Hannah cannot get a direct flight from Heathrow to Sydney.

- 1 What is the difference in price between the most expensive flight and the cheapest flight to Sydney?
- 2 They plan to leave Heathrow on 20 August.
What date and time would each flight get them into Sydney?
- 3 They need to arrive back into Heathrow on 3 September.
What date and times could they get a flight back?
- 4 They want to leave Heathrow on 20 August and be back in Heathrow on 3 September.
What combination of flights gives them:
 - a the longest time in Sydney
 - b the least time in Sydney?

Fare (£)	Heathrow to Sydney			Sydney to Heathrow		
	Departure	Arrival (local time)	Approximate journey time (hours)	Departure	Arrival (local time)	Approximate journey time (hours)
830	21:00	10:00	26	09:30	15:30	41
890	16:00	18:00	39	10:50	05:50	30
1092	19:15	07:15	25	14:00	03:00	24

Answers

- 1 34 m
- 2 22
- 3 a £240 b £130
- 4 31
- 5 a 1080.25, 950.25, 700.25, 952.96, 952.96
b debit is money taken out; credit is money put in
- 6 a 19 759 m b 2063 m
- 7 a

Item	Buying price (£)	Selling price (£)	Profit (£)
Silver dish	59	41	-18
Umbrella	33	47	+14
Toy car	45	55	+10
Total	137	143	+6

b

Item	Buying price (£)	Selling price (£)	Profit (£)
Silver dish	38	55	+17
Umbrella	49	85	+36
Toy car	110	58	-52
Total	197	198	+1

c The Red team won by £5.00.

d

	Red team without the bonus buy	Red team with the bonus buy
Blue team without the bonus buy	Red won	Red won
Blue team with the bonus buy	Blue won	Red won

- 8 a -5 b -27 c 56 d -132 e 63 f -24
g -60 h 96 i 23 j -1400 k 300 l 720

Answers

A Where shall we go?

- 1 San Francisco
- 2 1 hour
- 3 3 hours
- 4 5pm
- 5 4:30am

B How shall we get there?

- 1 £262
- 2 10:00, 22nd, 18:00 23rd, 07:15, 22nd
- 3 09:30, Sept 1st, 10:50, Sept 1st, 14:00, Sept 1st
- 4 a 19:15 from Heathrow and 1400 from Sydney on Sept 1st
b 16:00 from Heathrow and 0930 from Sydney on Sept 1st

Ready to progress?



I can work out the term-to-term rule for a sequence.
I can work out the operation in a function machine that uses one rule, when I am given the inputs and outputs.
I know the square numbers up to 15×15 .



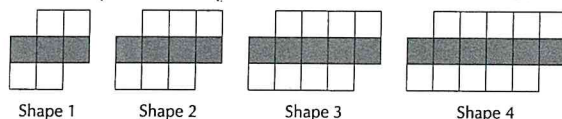
I can work out the inputs when given the outputs and the rules for an inverse function machine.
I can work out any term in a sequence, given the first term and the term-to-term rule.
I can recognise and work out the sequence of triangular numbers.
I can investigate the patterns and connections within the square and triangular numbers.
I can recognise and work out the numbers in the Fibonacci sequence and in Pascal's triangle.
I can work out any term in a sequence, given the n th term.



I can work out the operations in a function machine that uses more than one rule, when I am given the input and output values.
I can work out the n th term for any linear sequence.
I can investigate a given rule and reach a conclusion about whether it always works.

Review questions

- 1 Here is a sequence of shapes made with blue and white tiles.



The number of blue tiles = the shape number + 2
The number of white tiles = $2 \times$ the shape number + 2

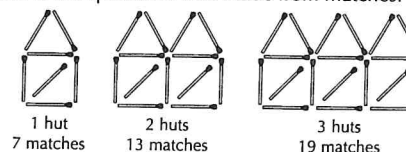
- How many blue tiles will there be in shape number 8?
 - How many white tiles will there be in shape number 35?
 - How many tiles altogether will there be in shape number 50?
 - Write down the missing numbers from this sentence:
The total number of tiles = $\dots \times$ the shape number + $\dots$
- 2 In each of these function machines, the two rules can be replaced with one rule.
Work out what this is for each machine. The first one has been done for you.

a $\rightarrow +2 \rightarrow -7 \rightarrow = \rightarrow -5 \rightarrow$ b $\rightarrow -7 \rightarrow +3 \rightarrow = \rightarrow ? \rightarrow$
c $\rightarrow \times 9 \rightarrow \times 2 \rightarrow = \rightarrow ? \rightarrow$ d $\rightarrow \times 4 \rightarrow \div 8 \rightarrow = \rightarrow ? \rightarrow$

- Write down the next two numbers in the sequence below.
93, 85, 77, 69, 61, ..., ...
 - Write down the next two numbers in this sequence.
1, 2, 4, 7, 11, 16, ..., ...
 - What two numbers do both sequences have in common, if they are continued?
- Maisy saves £5 each week for 10 weeks. How much will she have after five weeks?
 - Daisy saves £1 the first week, £2 the second week, £3 the third week, £4 the fourth week, and so on for 10 weeks. Copy and complete the table.

Week	1	2	3	4	5	6
Amount saved	£1	£2	£3	£4	£5	£6
Total amount saved	£1	£3				

- Work out who will have more money at the end of 10 weeks.
- 5 This is a sequence of huts made from matches.

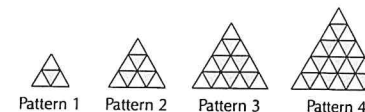


- Work out the rule for the n th term (the number of matches in the n th hut).
- Work out how many matches will be needed for the 30th hut.
- I use 91 matches to make a sequence of huts. How many huts do I make?
- I have 200 matches. What is the largest number of huts I can make?
How many matches will be left over?

- 6 Work out the n th term for each sequence.

a 5, 14, 23, 32, 41, ..., ... b 12, 10, 8, 6, 4, ..., ... c 3.5, 4, 4.5, 5, 5.5, ..., ...

- 7 The patterns in this sequence are made from blue and yellow triangles.



- a Copy and complete this table.

Pattern (term) number	Number of blue triangles	Number of yellow triangles	Total number of triangles
1	1	3	4
2	3		
3			
4			

- Describe the sequence formed by the numbers of blue triangles.
- Describe the sequence formed by the numbers of yellow triangles.
- Describe the sequence formed by the total numbers of triangles.
- Work out the total number of triangles in Pattern 10.
- What is the total number of triangles in the n th pattern?

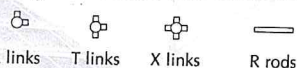
Mathematical reasoning

Valencia Planetarium

This is the planetarium in the city of Valencia in Spain. Many of the features of the building are based on a repeating sequence.

This key explains the diagrams on these pages.

Ladders and grids are made from combinations of:

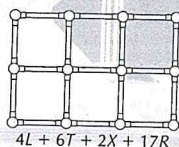


Each combination can be expressed as a rule, using letters.

Example

This grid is a combination of L links, T links, X links and R rods.

The combination can be written as the rule:
 $4L + 6T + 2X + 17R$



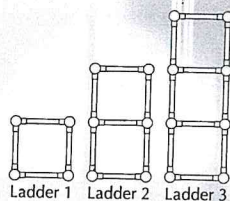
$4L + 6T + 2X + 17R$

1 Look at the ladders on the right.

a Use letters to write down a rule for each of them.

b Copy and complete this table.

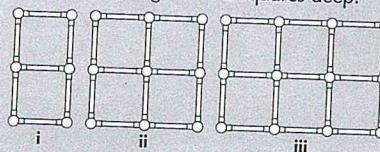
Ladder	L links	T links	R rods
1	4	0	4
2	4	2	7
3			
4			
5			



c Use letters to write down a rule for the links and rods in ladder 10.

d Use letters to write down a rule for the links and rods in ladder n .

2 These rectangles are 2 squares deep.



a Use letters to write down a rule for each of them.

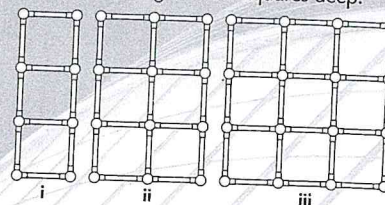
b Copy and complete this table.

Rectangle	L links	T links	X links	R rods
2 by 1	4	2	0	7
2 by 2	4	4	1	12
2 by 3				
2 by 4				
2 by 5				

c Use letters to write down a rule for the links and rods in a 2 by 10 rectangle.

d Use letters to write down a rule for the links and rods in a 2 by n rectangle.

3 These rectangles are 3 squares deep.



a Use letters to write down a rule for each of them.

b Copy and complete this table.

Rectangle	L links	T links	X links	R rods
3 by 1	4	4	0	10
3 by 2	4	6	2	17
3 by 3				
3 by 4				
3 by 5				

c Use letters to write down a rule for the links and rods in a 3 by 10 rectangle.

d Use letters to write down a rule for the links and rods in a 3 by n rectangle.

e Now write down a rule for the number of links and rods in a 4 by n rectangle.

4 a Use letters to write down a rule for each of these squares.

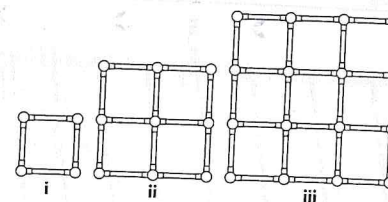
b Copy and complete this table.

Square	L links	T links	X links	R rods
1 by 1	4	0	0	4
2 by 2	4	4	1	12
3 by 3				

c Continue the table to work out the number of links and rods in a 7 by 7 square.

d Describe how the pattern of T links is building up.

e What type of special sequence do the numbers of X links make?



f Show that the number of rods in an n by n square is given by the n th term $2n(n+1)$.

g Use letters to write down a rule for the number of links and rods in an 8 by 8 square.

h Use letters to write down a rule for the number of links and rods in an n by n square.

Answers to Review questions

- 1 a 10 b 72 c 154 d Total = $3 \times$ shape number + 4
- 2 b -4 c $\times 18$ d $\div 2$
- 3 a 53, 45 b 22, 29 c 29, 37
- 4 a £25 b £6, £10, £15, £21 c Maisy has £50 and Daisy has £55.
- 5 a $6n + 1$ b 181 c 15 d 33, 1
- 6 a $9n - 4$ b $-2n + 14$ or $14 - 2n$ c $0.5n + 3$
- 7 a

Pattern Number	Number of blue triangles	Number of yellow triangles	Total number of triangles
1	1	3	4
2	3	6	9
3	6	10	16
4	10	15	25

Answers to Mathematical reasoning – Valencia Planetarium

- 1 a $4L + 4R$, $4L + 2T + 7R$, $4L + 4T + 10R$

b

Ladder	L links	T links	R Rods
1	4	0	4
2	4	2	7
3	4	4	10
4	4	6	13
5	4	8	16

c $4L + 18T + 31R$
d $4L + (2n - 2)T + (3n + 1)R$

- 2 a $4L + 2T + 7R$, $4L + 4T + X + 12R$, $4L + 6T + 2X + 17R$

b

Rectangle	L links	T links	X links	R Rods
2×1	4	2	0	7
2×2	4	4	1	12
2×3	4	6	2	17
2×4	4	8	3	22
2×5	4	10	4	27

c $4L + 20T + 9X + 52R$
d $4L + 2nT + (n - 1)X + (5n + 2)R$

- 3 a $4L + 4T + 10R$, $4L + 6T + 2X + 17R$, $4L + 8T + 4X + 24R$

b

Rectangle	L links	T links	X links	R Rods
3×1	4	4	0	10
3×2	4	6	2	17
3×3	4	8	4	24
3×4	4	10	6	31
3×5	4	12	8	38

c $4L + 22T + 18X + 73R$
d $4L + (2n + 2)T + (2n - 2)X + (7n + 3)R$
e $4L + (2n + 4)T + (3n - 3)X + (9n + 4)R$

- 4 a $4L + 4R$, $4L + 4T + X + 12R$, $4L + 8T + 4X + 24R$

b c

Square	L links	T links	X links	R Rods
1×1	4	0	0	4
2×2	4	4	1	12
3×3	4	8	4	24
4×4	4	12	9	40
5×5	4	16	16	60
6×6	4	20	25	84
7×7	4	24	36	112

d Starts at 0 and adds 4 each time
e Square numbers
f e.g. for $n = 7$
 $2 \times 7 \times (7 + 1) = 14 \times 8 = 112$, so the rule works
g $4L + 28T + 49X + 144R$
h $4L + (4n - 4)T + (n - 1)^2X + 2n(n + 1)R$

Ready to progress?



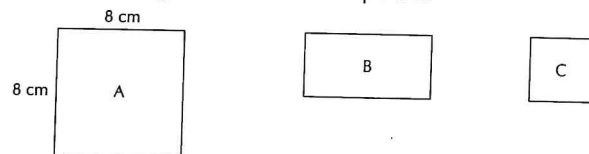
I can use the formula $A = lw$ to work out the area of a rectangle.
I can work out the perimeter of a compound shape.



I can work out the area of a compound shape.
I can work out the area of a triangle, of a parallelogram and of a trapezium by using the appropriate formula.
I can use the formula $S = 2lw + 2lh + 2wh$ to work out the surface area of a cuboid.
I can use the formula $V = lwh$ to work out the volume of a cuboid.

Review questions

- 1 The diagram shows square A. Fold square A in half to make rectangle B. Then fold rectangle B in half to make square C.

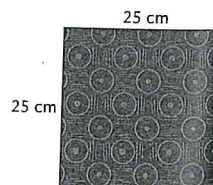


Copy and complete the table below, to show the area and perimeter of each shape.

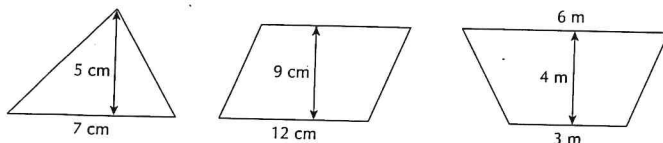
	Area	Perimeter
Square A	... cm ²	... cm
Rectangle B	... cm ²	... cm
Square C	... cm ²	... cm

- PS 2 A rectangle has an area of 48 cm².
The length is three times the width.
Work out the length of the rectangle.

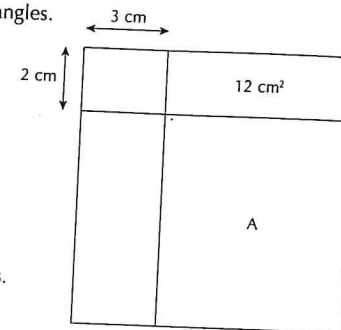
- 3 A shop sells square carpet tiles, as shown opposite.
The floor of a rectangular room is 3 m by 1.5 m.
How many tiles are needed to carpet the floor?



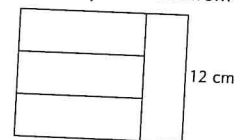
- 4 Work out the area of each shape.



- 5 The diagram shows a square, cut into four rectangles. The area of one of the rectangles is shown. Work out the area of the rectangle marked A.

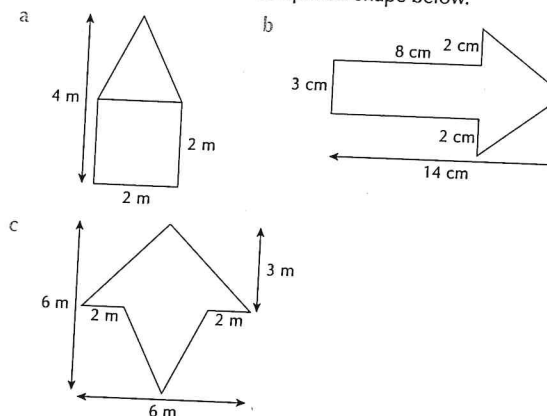


- PS 6 This shape is made from four identical rectangles.

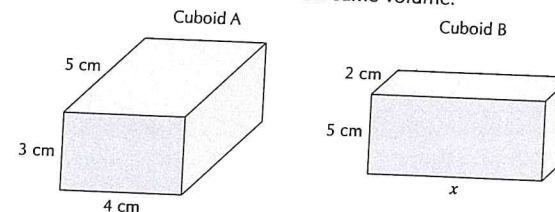


Work out the area of the shape.

- PS 7 Work out the area of each compound shape below.



- 8 The two cuboids A and B have the same volume.



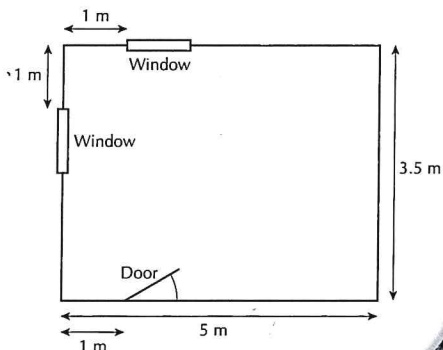
- a Work out: i the surface area ii the volume of Cuboid A.
b Work out the value of the length marked x on Cuboid B.

Problem solving

Design a bedroom

1 This is a sketch plan for a bedroom.

- What is the perimeter of the bedroom?
- What is the area of the bedroom?
- One square metre of carpet costs £32.50. How much does it cost to carpet the bedroom?



2 Posters cost £6.99 each.

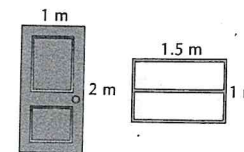
- How many posters can you buy for £50?
- How much money will you have left over?



3 These are sketches of the door and one of the windows.

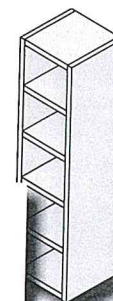
The height of the bedroom is $2\frac{1}{2}$ m.

- What is the total area of all four walls?
- A one-litre tin of paint covers 12 m^2 . What is the minimum number of tins required to paint the walls?



4 Ellie wants to buy a storage unit for her collection of DVDs, for her bedroom.

- This unit is 1 m high, 21 cm wide and has a depth of 17 cm. Work out the volume of the unit.
- Ellie wants to store her collection of DVDs in this unit. The thickness of the wood in the unit is 1.5 cm. The thickness of each DVD is 1.5 cm. What is the maximum number of DVDs she can store?



Answers

- 1 **A** 64 cm^2 , 32 cm ; **B** 32 cm^2 , 24 cm ; **C** 16 cm^2 , 16 cm
- 2 12 cm
- 3 72
- 4 **a** 17.5 cm^2 **b** 108 cm^2 **c** 18 m^2
- 5 42 cm^2
- 6 192 cm^2
- 7 **a** 6 m^2 **b** 45 cm^2 **c** 12 m^2
- 8 **a** 94 cm^2 **ii** 60 cm^3 **b** 6 cm

Answers

- 1 **a** Perimeter = 17 m **b** Area = 17.5 m^2 **c** Cost of carpet = $\pounds 568.75$
- 2 **a** 7 posters **b** $\pounds 1.07$ is left over
- 3 **a** Door wall = 10.5 m^2 , short window wall = 7.25 m^2 , long window wall = 11 m^2 , plain wall = 8.75 m^2 Total area of walls = 37.5 m^2
b 4 tins of paint
- 4 **a** $35\,700 \text{ cm}^3$ **b** 60